

## Detrended Fluctuation Analysis of Climate and Seismic Data: Examples from Bulgarian Data

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**Abstract.** In the last two decades the Detrended Fluctuation Analysis (DFA) method has been extensively applied as a tool to quantify long-term correlations in climate and seismic time series. DFA was also applied to Bulgarian climate and earthquake records. Here we review published results from application of DFA to Bulgarian climate and seismic data and we present some new results on persistence in earthquake magnitude time series in natural time for a Bulgarian earthquake catalog. We emphasize on the finding that the degree of long-term correlations in Bulgarian surface temperature data exhibit complex dependence on the topography and proximity to the Black Sea coast. This finding is somewhat surprising, since it contradicts some previous results of other authors indicating lack of significant influence of the distance to continental coastlines on the long-term dependence in climate records. We hypothesize that our finding is related to complex interplay of maritime influence and topography of the region. For a Bulgarian earthquake catalogue we observe that the interoccurrence intervals between successive earthquakes are not independent but exhibit strong and threshold independent persistence. For the same catalogue we also observe strong persistence of earthquake magnitudes in natural time.

### 1 Introduction

In the recent years the Detrended Fluctuation Analysis (DFA) method [1] established itself as the preferred tool for quantification of long-term correlations in empirical time series. The DFA method provides robust estimates of persistence in nonstationary time series [2–6]. Further, it has been shown that the DFA method performs very well at low frequencies [7] which is extremely important when we are interested in the long-term correlations. Traditional power spectrum and autocorrelation function methods, as well as the Hurst rescaled range analysis [8] have certain shortcomings when estimating long-term correlations in short, noisy and nonstationary time series.

In the last two decades the DFA method has been extensively applied to quantify long-term correlations in climate [7, 9–18] and seismic time se-

ries [19–25]. Here we outline some of the most important results on applications of DFA to climate and seismic records. We also review results from application of DFA to Bulgarian climate and seismic data. We present some new results on long-term correlations analysis of a Bulgarian earthquake catalogue.

The outline of the paper is as follows. In Section 2 we present the DFA method. In Section 3 we review some of the most important results on persistence in climate time series and we present examples from Bulgarian climate data. In Section 4 we consider applications of the DFA method to seismic time series and we present examples from Bulgarian seismic data. In Section 5 we summarize our findings.

## 2 The DFA Method

The DFA method [1] consists of the following steps: (i) first the studied time series  $S_i$  is integrated to construct the profile

$$Y(k) = \sum_{i=1}^k (S_i - \langle S \rangle),$$

where  $\langle S \rangle$  is the mean value of the investigated time series over the period we consider; (ii) the profile  $Y(k)$  is divided into consecutive segments of length  $n$  and the local trend in each segment is fitted with a least-squares polynomial fit; (iii) the profile  $Y(k)$  is detrended by subtracting the local polynomial trend in each segment of length  $n$ , and the root mean square fluctuation  $F(n)$  for the detrended profile is calculated. For order- $l$  DFA (DFA-1 if  $l = 1$ , DFA-2 if  $l = 2$ , etc.) a polynomial function of order  $l$  is applied to fit the local trend in each segment of the profile  $Y(k)$ ; (iv) this procedure is repeated for different scales  $n$ . A power-law relation  $F(n) \sim n^\alpha$  indicates the presence of scaling in the studied time series. Thus the fluctuations in  $S_i$  can be characterized by the scaling exponent  $\alpha$ , a self-similarity parameter that quantifies the fractal power-law correlation properties of the signal. To ensure sufficient statistics when calculating  $F(n)$  for large box sizes  $n$ , and thus a more accurate estimate of the scaling exponent  $\alpha$  at large time scales, the maximum box size should be less than  $n = N/4$ , where  $N$  is the length of the time series. To increase additionally the statistics at large time scales "sliding window" version of DFA may be applied, removing the polynomial trend in each overlapping window.

The scaling exponent  $\alpha$  is related to the autocorrelation function exponent  $\gamma$  ( $C(n) \sim n^{-\gamma}$  when  $0 < \gamma < 1$ ) and to the power spectrum exponent  $\beta$  ( $S(f) \sim 1/f^\beta$ ) by  $\alpha = 1 - \gamma/2 = (\beta + 1)/2$  [26, 27]. A value of  $\alpha = 0.5$  indicates that there are no correlations and the signal is uncorrelated (white noise). If  $\alpha < 0.5$  the signal is said to be *anti-correlated*,

meaning that large values are more likely to be followed by small values. If  $\alpha > 0.5$  the signal is correlated and exhibits persistent behavior, meaning that large values are more likely to be followed by large values and small values by small values. The higher the value of  $\alpha$ , the stronger the correlations in the signal.

### **3 Long-Term Correlations in Climate Records**

Koscielny-Bunde et al. (1998) [9] analyzed 14 daily temperature records from Europe, North America and Australia, and found long-term correlations in the temperature fluctuations characterized by an universal power-law with scaling exponent  $\alpha \approx 0.65$ . This scale invariant behavior persists up to several years and even decades. Subsequent studies have demonstrated that over the oceans and for small island stations temperature fluctuations exhibit stronger correlations [10–12], and that the long-term correlations depend also on the altitude of the station [7, 14] with lower values of the scaling exponents indicating weaker persistence for temperature records from higher altitudes. [12] studied larger surface temperature database and found long-term correlations with scaling exponent  $\alpha \approx 0.65$  only for temperature records from coastal and transitional regions, while over the interior of the continents they observed approximately uncorrelated behavior. Later, in a study of temperature fluctuations over an even denser set of points covering the continent of Australia Pattantyús-Ábrahám et al. [15] demonstrated that the scaling exponent characterizing persistence depends on the latitude. In a comprehensive study Kiraly et al. [17] analyzed thousands of temperature records and observed long-term correlations with complex spatial clustering of the scaling exponents characterizing persistence. Long-term power-law correlations were also found in records of other weather elements such as atmospheric pressure, relative humidity and the diurnal temperature range [14], as well as in records of the Southern Oscillation Index [16].

In a recent paper [28] we studied long-term correlations in 22 temperature records from 11 coastal and 11 Bulgarian weather stations. We found that coastal stations exhibit strong long-term persistence with scaling exponents typical of small islands and sea surface temperature persistence. Inland stations also exhibit long-term correlations but with lower scaling exponents (Figure 1). Further, we found that the contrast in the correlation properties between coastal and inland regions is greater for south Bulgaria than for north Bulgaria, and hypothesized that this finding is related to the topography difference between northern and southern parts of the country. To some extent, our finding contradicts previous results indicating lack of distance to coast dependence of the strength

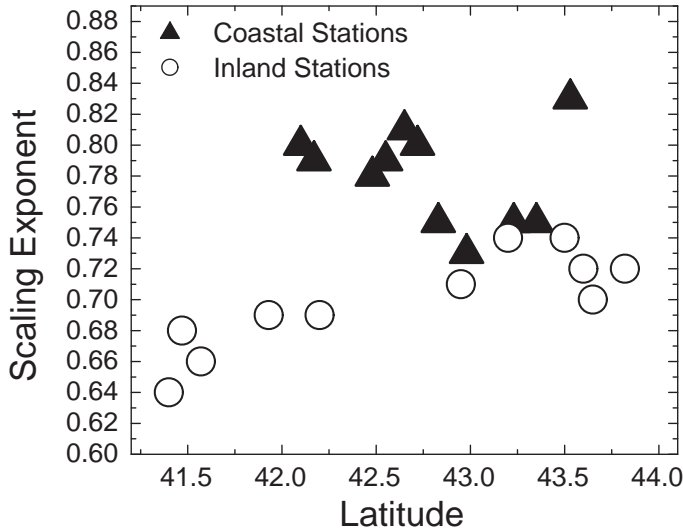


Figure 1: Values of the scaling exponent characterizing long-term correlations obtained for 11 Bulgarian coastal stations and 11 Bulgarian inland stations. Filled triangles: coastal stations. Empty circles: inland stations. Coastal stations exhibit stronger long-term correlations. Note the higher differences between coastal and inland stations for south Bulgaria than for north Bulgaria. Adapted from [28].

of long-term persistence [11, 17]. However, these studies are based on a large number of temperature records from locations with highly varied geographic characteristics. Our results indicate that in certain regions with specific topography distance to coast may be significant for the strength of the long-term persistence.

DFA was used as a tool to identify and quantify nonlinearity in time series by a method proposed by Ashkenazy et al. [29], which consists of the following steps. First, time series are decomposed into two components: (1) series of magnitudes defined as the absolute value of the temporal increments, and (2) series of signs of the consecutive increments. When the magnitude (called also volatility) series exhibit long-term persistence, the original time series is nonlinear. This approach was initially applied to surface temperature records by Govindan et al. [30]. They reported long-term positive correlations in magnitude series of daily mean temperature from 10 sites around the globe, with scaling exponents characterizing persistence about 0.6 for all 10 records. Later, we studied magnitude and sign correlations in daily temperature records from station Sandanski [31] and we found that for both minimum temperature and

maximum temperature, the magnitude series exhibit positive long-term correlations with scaling exponent estimates of about 0.6, in agreement with the findings of Govindan et al. [30]. However, in a more comprehensive study Bartos and Janosi [18] studied volatility in thousands temperature records, and reported positive long-term correlations for the studied temperature records but with a site dependent scaling exponent. We also studied volatility in diurnal temperature range and sunshine duration records from station Sandanski [32] and found that they exhibit no long-term correlations, indicating linear behavior.

#### **4 Long-Term Correlations in Seismic Records**

In the recent years the DFA method and similar scaling techniques were widely used to characterize long-term correlations in two types of earthquake data: (1) time series of interevent intervals between consecutive earthquakes and (2) earthquake magnitude time series in the so called “natural time”, i.e. the first earthquake is  $i = 1$ , the second earthquake is  $i = 2$ , and so on.

Most of the studies concentrated on interevent intervals between consecutive earthquakes. Lennartz et al. [23] demonstrated presence of persistent power-law correlations in the temporal behavior of Northern and Southern California seismicity. Telesca et al. [20] studied the spatio-temporal behavior of the Southern California seismicity and reported threshold magnitude dependent long-term correlations with diminishing persistence for higher threshold magnitude. Same authors [21] reported threshold dependence as well as spatial variability of the scaling behavior of Central Italy seismicity. Contrary, Enescu et al. [19] reported lack of significant temporal correlations in the interoccurrence intervals between earthquakes for a catalogue of Vrancea seismicity.

In a recent paper [33] we studied scaling behavior of interevent intervals for a Bulgarian seismicity catalog published by the Bulgarian Academy of Sciences [34]. We applied the DFA-1 and DFA-2 methods and found evidence for long-term power-law correlations with scaling exponent  $\alpha \approx 0.75$  indicating strong persistence (Figure 2). The scaling exponent is threshold independent over a range of threshold magnitudes between 3.0 and 3.4. Currently, it is difficult to interpret the seismotectonic relevance of this finding. We note that typically the strength of long-term persistence in interevent intervals time series decreases with increasing the threshold magnitude [20, 23, 35]. However, in certain regions the scaling exponent characterizing persistence is threshold independent or even increases with the increase of threshold magnitude [21].

Recently, there is growing interest in the temporal correlations between earthquake magnitudes, analyzed in natural time [36]. Lennartz et al.

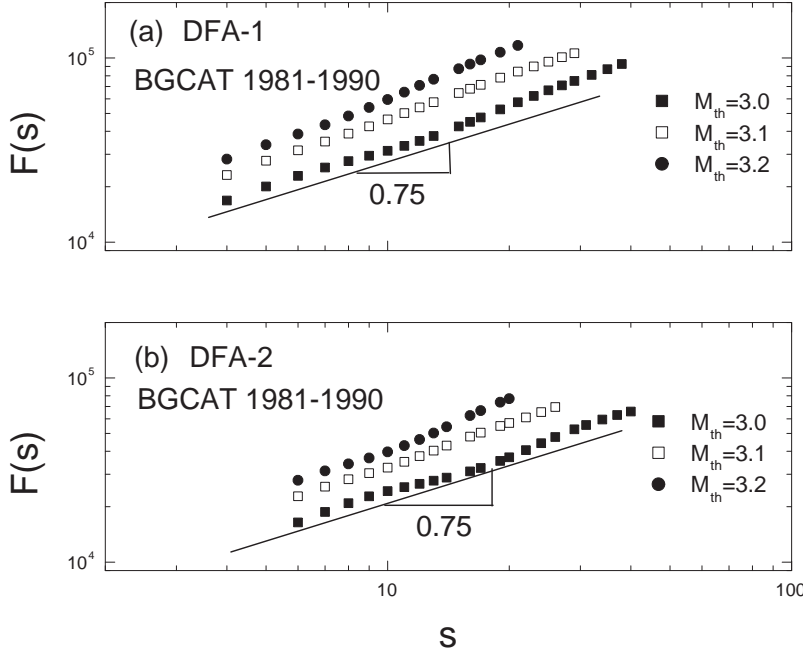


Figure 2: Root mean square fluctuation,  $F(n)$ , obtained using DFA-1 (a) and DFA-2 (b) for interoccurrence intervals between earthquakes from the Catalogue of Bulgarian earthquakes for the period 1981-1990 [34]. Presented are scaling curves for earthquakes with magnitudes exceeding different threshold values. Records for all threshold magnitudes exhibit positive long-term correlations with scaling exponents  $\alpha \approx 0.75$  (modified from [33]).

[24] analyzed long-term persistence in earthquake magnitude time series from Northern and Southern California and obtained estimates of the scaling exponent between 0.6 and 0.75 indicating moderate persistence. Varotsos et al. [25] studied long-term correlations in earthquake magnitude time series from Japan and found a characteristic pattern of temporal evolution preceding certain large earthquakes.

Here, we study long-term persistence in earthquake magnitudes from the same earthquake catalogue we analyzed for long-term correlations in interevent intervals [33]. For time scales up to about 40 events we obtain an estimate of the scaling exponent  $\alpha \approx 0.65$  indicating moderate persistence (Figure 3). This result is in agreement with the estimates obtained by Lennartz et al. [24] for California. For shorter time scales (up to several events) we observe even stronger persistence.

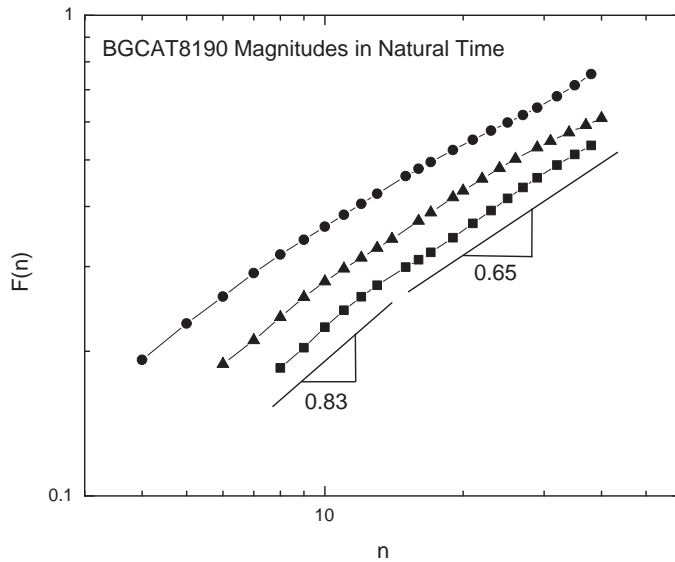


Figure 3: Root mean square fluctuation,  $F(n)$ , obtained using DFA-1, DFA-2 and DFA-3 for earthquake magnitudes in natural time from the Catalogue of Bulgarian earthquakes for the period 1981-1990 [34]. For time scales up to about 40 events we observe moderate persistence with scaling exponent  $\alpha \approx 0.65$ . On shorter time scales (up to several events) we observe stronger persistence with scaling exponent  $\alpha \approx 0.8$ .

## 5 Summary and Conclusions

We presented results on long-term correlations in (i) surface climate records from Bulgarian weather stations, and (ii) earthquake records from a Bulgarian earthquake catalogue. We compared the obtained values of the scaling exponents with findings of other authors for long-term dependence in climate and seismic records from different regions of the globe.

We emphasize on our finding that the scaling exponents characterizing long-term persistence in Bulgarian climate data depend on the topography and proximity to the Black Sea coast. This finding is somewhat surprising, since it contradicts some previous results of other authors indicating lack of significant influence of the distance to continental coastlines on the long-term dependence in climate records. We commented on the possible explanation of this apparent contradiction.

For a Bulgarian seismicity catalogue we found that the interoccurrence intervals between consecutive earthquakes are not independent but ex-

hibit strong and threshold independent long-term power-law correlations. For the same earthquake catalogue we also observe moderate long-term persistence of earthquake magnitudes in natural time, in agreement with results of other authors for different regions of the globe.

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